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Discussion on:

Reliability evaluation of reinforced concrete beams

R. Lu, Y. Luo and J.P. Conte
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1. The aim of the discussion

In the paper under discussion the authors give a comprehensive analysis of reliability of RC beams designed under the provisions of ACI Building Code and obtain many important results. The implication of their analysis is that ACI Code 318-89 gives a uniform reliability for the investigated design situations. All component reliability indices vary from 3.2 to 4.2.

The aim of this discussion is to show that in some cases the probability of brittle failure is rather high, reliability is inadequate and reliability indices go well below 3.2. The reasons for these phenomena are revealed and some measures to remedy the situation are recommended.

In what follows the paper under discussion will be termed "the paper".

2. Initial data

Let us consider flexural strength of the beams. Three modes of bending failure are possible depending on whether the beam is lightly, moderately or over-reinforced. To distinguish between the modes the following limit-state functions are used in the paper:

$$g_1 = A_s - \frac{200}{f_y} b_w d, \tag{1}$$

$$g_2 = A_s - \frac{0.85\beta_1 f_c'}{f_y} \frac{87000}{87000 + f_y} b_w d. \tag{2}$$

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Equations (1) and (2) correspond to Eqs. (16) and (17) in the paper. Here

A_s = area of tension reinforcement (in^2);
b_w, d, h = width, effective depth and height of beam cross section (in);
f'_c, f_y = compressive strength of concrete, yield strength of steel (ksi);
β_1 = the ratio of the depth of stressed block in the compression zone to the distance between the outside compression surface and the neutral axis (here as well as in the paper $\beta_1 = 0.85$).

In the paper conditions $g_1 < 0$ and $g_2 > 0$ hold for light- and over-reinforcement, respectively. Otherwise (i.e., if $g_1 > 0$ and $g_2 < 0$) the beam is moderately reinforced.

The limit-state functions for lightly, moderately and over-reinforced beams are, respectively, as follows:

$$g_3 = B_f\left(1.25 b_w h^2 \sqrt{f'_c}\right) - M, \tag{3}$$

$$g_4 = B_f A_s f_y \left(d - \frac{A_s f_y}{1.7 f'_c b_w}\right) - M, \tag{4}$$

$$g_5 = B_f\left(\frac{1}{3} b_w d^2 f'_c\right) - M. \tag{5}$$

Equations (3)–(5) correspond to Eqs. (18)–(20) in the paper. Here

B_f = factor characterizing flexural model uncertainty;
M = external bending moment.

For generality let us divide both sides of Eq. (2) by $b_w d$ and both sides of Eqs. (3)–(5) by $b_w d^2$. Then

$$g_2^r = \rho - \frac{0.85\beta_1 f'_c}{f_y} \frac{87000}{87000 + f_y}, \tag{6}$$

$$g_3^r = B_f\left(1.65\sqrt{f'_c}\right) - M^r, \tag{7}$$

$$g_4^r = B_f \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c}\right) - M^r, \tag{8}$$

$$g_5^r = B_f\left(\frac{1}{3} f'_c\right) - M^r. \tag{9}$$

Here $\rho = A_s/(b_w d)$ is reinforcement ratio and superscript r stands for relative values of limit-state functions and external moments. For the sake of definiteness it is assumed in Eqs. (3), (8) that $h = 1.15d$.

As was mentioned above, in the paper conditions $g_1 < 0$ and $g_2 > 0$ are used to distinguish between the cases of light- and over-reinforcement, respectively. Below we shall employ a similar condition $g_2^r > 0$ for over-reinforcement. To distinguish the light reinforcement another criterion instead of the condition $g_1 < 0$ will be used. The beam is lightly reinforced if its uncracked strength is greater than cracked strength. Therefore the condition $g_3^r > g_4^r$, or

$$1.65\sqrt{f_c'} > \rho f_y \left(1 - \frac{\rho f_y}{1.7 f_c'}\right) \tag{10}$$

holds for lightly reinforced beams.

3. Probability of brittle failure

In the paper only moderately reinforced beams with limit-state functions (4) are considered: it is correctly reasoned that the probabilities of light- and over-reinforcement are very small. However, the beam *initially* (deterministically) designed as moderately reinforced can be *actually* lightly or over-reinforced. Let us estimate probabilities of these events.

By way of example consider a beam with the following material strengths: $f_y = 40$ ksi; $f_c' = 4$ ksi. The beam is moderately reinforced if its reinforcement ratio satisfies the following conditions:

$$\rho_{\min} \le \rho \le \rho_{\max} \tag{11}$$

where $\rho_{\min} = 0.005$; $\rho_{\max} = \frac{3}{4}\rho_b = 0.037$. The balanced reinforcement ratio ρ_b is determined by the following formula:

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y} \frac{87000}{87000 + f_y}. \tag{12}$$

In the paper the yield stress of steal is represented by a beta distribution with mean value 48.8 ksi and c.o.v. = 0.107; the lower and upper bounds are, respectively, 33 and 62 ksi. Compressive strength of concrete is normally distributed with mean value 3.8 ksi and c.o.v. = 0.180. If material strengths are random values, then the balanced reinforcement ratio ρ_b is a random value too. Let us denote it by ρ_b^*. Using Eq. (6) probabilities $P(\rho > \rho_b^*) = P(g_2^{r*} > 0)$ and $P(\rho > \frac{3}{4}\rho_b^* = \rho_{\max}^*)$ have been determined (sign * stands for random values). Probability $P(\rho > \rho_b^*)$ is the probability that the beam initially designed as moderately reinforced with reinforcement ratio satisfying conditions (11) is actually over-reinforced. In much the same way probability $P(\rho > \frac{3}{4}\rho_b^*)$ is the probability that provisions of ACI Building Code for moderately reinforced beams are violated.

Calculations were performed using three approaches:

(1) Monte Carlo simulation with subsequent approximation of the results by Pearson's curves and numerical integration [1,2]; sample size was 5000 (this procedure is described in more detail in the next section).

(2) Crude Monte Carlo simulation with sample size 5000.

(3) Crude Monte Carlo simulation with sample size 15 000 (for some ρ values).

All results were in close agreement. They are presented in Table 1.

As can be seen from Table 1, the probabilities $P(\rho > \rho_b^*)$ and $P(\rho > \frac{3}{4}\rho_b^*)$ are very high for high ρ values and gradually go down as ρ values decrease. Thus, the probability of brittle

Table 1
Probabilities $P(\rho > \rho_b^*)$ and $P(\rho > \frac{3}{4}\rho_b^*)$

ρ	0.037	0.035	0.033	0.030	0.028	0.025	0.023	0.020	0.018	0.015	0.013	0.010
$P(\rho > \rho_b^*)$	0.543	0.469	0.384	0.255	0.184	0.094	0.053	0.016	0.007	0.001	0.0002	< 0.0002
$P(\rho > \frac{3}{4}\rho_b^*)$	0.874	0.829	0.768	0.650	0.554	0.396	0.285	0.138	0.071	0.016	0.006	0.0002

failure is fairly high even for relatively low ρ values: for example, $P(\rho > \rho_b^*) = 0.053$ for $\rho = 0.023$. The reason for this phenomenon is quite apparent. The mean value of concrete strength 3.8 ksi is rather low, less than $f_c' = 4.0$ ksi, and c.o.v. = 0.180 is rather high. As a result the probability that concrete strength will fall below f_c' is high (it equals 0.614). From Eq. (6) one can see that the probability of over-reinforcement increases as concrete strength decreases. In view of high probability of low concrete strength values the probability of over-reinforcement is high.

To estimate the probability of light reinforcement a similar methodology was used. In the course of Monte Carlo simulation condition (10) was checked for $\rho = \rho_{\min} = 0.005$. Three random variables $f_c'^*$, $\sqrt{f_c'^*}$, f_y^* were taken into account. Their distributions for $f_c' = 4$ ksi and $f_y = 40$ ksi are given in the paper. It turned out that probability of light reinforcement was very close to zero (less than 10^{-4}). Therefore in what follows the possibility of brittle failure due to light reinforcement will be neglected.

4. Reliability of beams

As is evident from the foregoing, two failure modes described by limit-state functions (8), (9) should be taken into account in the course of beam reliability analysis. FORM/SORM methods used in the paper can take into account only one limit-state function. Therefore it is difficult, if not impossible, to use the methods in the case of two limit-state functions. Below is given another approach. By way of example reliability of the beam with material properties $f_c' = 4$ ksi and $f_y = 40$ ksi is determined. Live-to-dead load ratio is assumed to be 2.5.

The initial data for calculations were prepared in the following way:

1. Specify reinforcement ratio ρ satisfying conditions (11).
2. Determine the nominal relative moment capacity of the beam M_n^r. Assume that the factorized relative external moment M_f^r equals ϕM_n^r (ϕ is a strength reduction factor, $\phi = 0.9$).
3. Take $M_{1f}^r = 0.25 M_f^r$ and $M_{2f}^r = 0.75 M_f^r$. Assume that M_{1f}^r and M_{2f}^r are factorized external moments produced by dead and live loads, respectively. Coefficients 0.25 and 0.75 correspond to live-to-dead load ratio 2.5 for unfactored loads.
4. Find unfactored moments: $M_1^r = 0.25 M_{1f}^r/1.4$; $M_2^r = 0.75\ M_{2f}^r/1.7$ (here 1.4 and 1.7 are load factors for dead and live loads, respectively).

5. Assume M_1^r and M_2^r to be mean values of the random moments M_1^{r*} and M_2^{r*} produced by dead and live loads, respectively. According to the data in the paper assume that M_1^{r*} is normally distributed with c.o.v. = 0.10 and M_2^{r*} fits a type 1 extreme value distribution with c.o.v. = 0.25.
6. In Eqs. (8), (9) assume that $M^r = M_1^{r*} + M_2^{r*}$, $f_y = f_y^*$, $f_c' = f_c'^*$; $B_f = B_f^*$, $\sqrt{f_c'} = \sqrt{f_c'^*}$, where f_y^*, $f_c'^*$, B_f^*, $\sqrt{f_c'^*}$ are random values; parameters of their distributions are given in the paper.

Perform calculations in the following order:

1. Using Monte Carlo simulation obtain a set of realizations of random variables f_y^*, $f_c'^*$, B_f^*, $\sqrt{f_c'^*}$, M_1^{r*}, M_2^{r*}.
2. Check condition (6) to determine whether the beam is moderately or over-reinforced.
3. Choose the corresponding limit-state function among (8), (9) and calculate its value g^r.
4. Perform steps 1 to 3 m times. As a result obtain m values $g_1^r, \ldots, g_m^r$.
5. Fit an appropriate Pearson's curve $y(z)$ to describe probability density functions of g^r values.
6. Calculate reliability of the beam R by numerical integration:

$$R = \int_0^{+\infty} y(z)\, \mathrm{d}z. \tag{13}$$

All calculations were performed with sample size $m = 5000$ and for some cases were checked by crude Monte Carlo simulation with sample size 15 000. Reliabilities obtained by the two methods were in close agreement. To compare the results with those obtained in the paper the calculations were also performed using one failure mode, corresponding to the moderately reinforced beam and described by failure function (8).

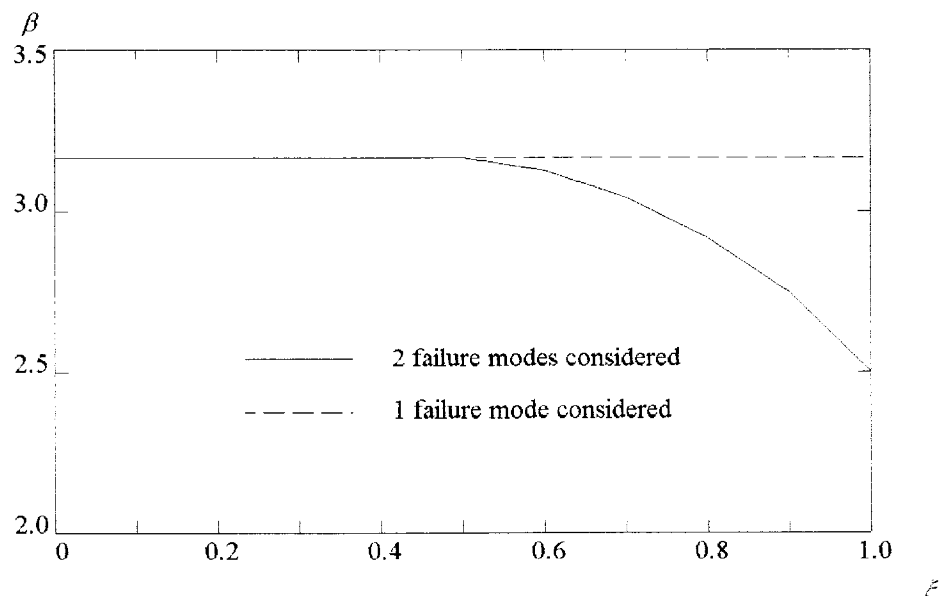

Fig. 1.

Calculation results are presented in Fig. 1. Here the reliability indices β are plotted vs. ξ. The ξ values are associated with the reinforcement ratio ρ in the following way:

$$\xi = \frac{\rho - \rho_{\min}}{\rho_{\max} - \rho_{\min}}. \tag{14}$$

The β values are determined for eleven ξ values: $\xi = 0, 0.1, 0.2, \ldots, 1$.

The obtained results are in complete agreement with the values of probabilities $P(\rho > \rho^*_{\max})$ discussed above. As can be seen from Fig. 1, if two failure modes are taken into account (solid line), the reliability index $\beta = 3.15$ remains unchanged for $0 \le \xi \le 0.5$ because in this case for the most part limit-state function (8) is used and it gives the highest g^r values. As ξ increases from 0.5 to 1, the probability $P(\rho > \rho^*_{\max})$ increases as well (see Table 1). As a result in the course of Monte Carlo simulation in increasing number of cases limit-state function (9) is used and this function gives lower g^r values in comparison with limit-state function (8). For $\xi = 1$ ($\rho = \rho_{\max}$) reliability index β drops to 2.5.

Thus, reliability of beams is low if reinforcement ratio is high (ρ is close to $\rho_{\max}$). Under these conditions moment capacity of the beam is governed predominantly by concrete; reinforcement does not contribute to the beam moment capacity (see Eq. (9)). Therefore reliability is low.

In case of low and average reinforcement ratio ($0 \le \xi \le 0.5$, see Fig. 1) the beam is actually moderately reinforced, i.e., the probability of over-reinforcement is very small. Flexural strength of such beam is governed by both materials—concrete and steel (see Eq. (8)). The materials behave as though they support each other.

Assume, for example, that concrete strength is low. Then if strength of reinforcement is sufficiently high, the depth of concrete compression zone increases and the beam can support the external moment with a reduced value of the arm of the internal couple. Similarly, if strength of reinforcement is low, but strength of concrete is sufficiently high, the depth of concrete compression zone decreases and the beam can support the external moment with a larger value of the arm of the internal couple.

Maximum reliability index $\beta = 3.15$ coincides very closely with reliability index $\beta = 3.2$ obtained in the paper for live-to-dead load ratio 2.5 (see Fig. 3 in the paper) as well as with the reliability index calculated taking into account one failure mode (the dashed line in Fig. 1). However, if only one failure mode is considered, the reliability index $\beta = 3.15$ remains constant on the whole interval $0 \le \xi \le 1$. From this it follows that the failure mode related to over-reinforcement must not be neglected.

To gain a better understanding of the matters under discussion, all above calculations were carried out for the beam with material strengths $f'_c = 4$ ksi; $f_y = 40$ ksi: in this case the effects are more pronounced. In the paper two other cases were considered: $f'_c = 3$ ksi; $f_y = 40$ ksi and $f'_c = 4$ ksi; $f_y = 60$ ksi. These cases were checked too. If only one failure mode corresponding to moderate reinforcement was considered, the obtained results were in close agreement with those given in the paper.

Fig. 1 indicates that reliability of the beams designed under the provisions of ACI Building Code is non-uniform and inadequate for ρ values close to $\rho_{\max}$. The problem arises how to remedy the situation.

5. Recommendations

In my opinion, matters can be straightened out by the following measures.

As was mentioned above, in case of over-reinforcement inadequate reliability arises from low strength of concrete, which in its turn is directly related to evaluation and acceptance rules. Under the provisions of ACI Building Code the concrete is considered acceptable if two criteria are met:

(1) No single test strength shall be more than 500 psi below the specified compressive strength f'_c.

(2) The average of any three consecutive test results must equal or exceed the specified compressive strength, f'_c.

The second criterion implies that the minimum required average compressive strength of concrete is equal to the specified compressive strength, f'_c. Consider the concrete in a batch. Assume that concrete strength is normally distributed with mean value f'_c. According to the second criterion, the concrete is accepted. In this batch the probability $P(f'^*_c > f'_c)$ that actual concrete strength f'^*_c exceeds f'_c is very low and equals 0.5. In practice this exceedance probability is even lower; for concrete with $f'_c = 4.0$ ksi and c.o.v. $= 0.18$ the exceedance probability $P(f'^*_c > f'_c)$ is only 0.386.

Several years ago similar drawbacks in the Russian Code for RC structures design came to light [1]. To remedy the situation it was decided to change the definitions of characteristic and design strengths of concrete. Previously characteristic strength B_n was specified with exceedance probability 0.95. Design strength R_d was defined as the ratio $R_d = B_n/\gamma_c$, where γ_c is a partial safety factor ($\gamma_c > 1$). To get rid of the cases with inadequate reliability additional requirements on characteristic B_n and design R_d strengths of concrete were imposed: probability that concrete strength exceeds B_n and R_d should be not less than 0.95 and 0.9986, respectively, with partial safety factor γ_c being unchanged. Then the control procedures in the State Standard GOST 18105-86 "Concrete. Rules for Acceptance Control" were changed to meet these requirements. This procedure is described in detail in [3,4].

It is appropriate to consider a possibility to apply a similar approach to ACI Code 318-89. Specified compressive strength of concrete f'_c can be defined with a certain exceedance probability. Then the rules for acceptance control of concrete can be changed in such a way as to satisfy this definition. In this case the minimum average required compressive strength of concrete will, of course, exceed f'_c.

By these means the cases with inadequate reliability are eliminated. However, excessive reliability can appear in some cases (e.g., for $0 \le \xi \le 0.5$, see Fig. 1). In such an event a material combination factor [1] can be introduced to regulate reliability.

The material combination factor is an additional partial safety factor. It is similar to load factors. It takes into account low probabilities of simultaneously low values of strength of several materials, when the materials behave as if they support each other. By comparison, load factors take into account low probabilities of simultaneously high values of several loads. With the material combination factor a uniform reliability can be achieved.

This approach can be applied not only to beams, but to other structures as well.

6. Conclusion

From the above discussion the following conclusions related to flexural strength of RC beams can be drawn:

1. Reliability of RC beams designed under the provisions of ACI Building Code is non-uniform and changes substantially with reinforcement ratio ρ. The lowest values of reliability occur for beams with ρ values close to $\frac{3}{4}\rho_b$.
2. For the above case the probability of brittle failure is rather high.
3. In the course of reliability analyses of RC beams two failure modes, corresponding to moderate and over-reinforcement should be taken into account.
4. For investigated cases the probability of brittle failure due to light reinforcement is very small and therefore can be neglected.
5. To decrease the probability of brittle failure due to over-reinforcement the rules for acceptance control of concrete can be changed.
6. To achieve a uniform reliability a material combination factor can be used.

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Authors' reply

The writers want to thank Professor Krakovski for his valuable comments and discussion of their paper. His contribution further enhances the understanding of the subject matter. In the interest of further clarification, the writers would like to offer the following comments in reply.

The reliability analysis of RC beams of rectangular cross sections designed according to the ACI Building Code, but which can actually fail in brittle mode requires additional statistical data to characterize the model uncertainty factor (B_f) corresponding to the brittle bending failure mode. As mentioned by the discusser, a reliability analysis accounting for both the ductile and brittle bending failure modes necessitates a system reliability approach. The curves in Fig. 1 of the discussion are produced by varying the cross-section dimensions and the relative amount of concrete and reinforcement steel at a fixed ratio of the moments due to dead and live loads ($M_2^r/M_1^r = 2.5$). This moment ratio is related to the live-to-dead load ratio, $\theta = L/D$, used in the paper through

$$\theta = \frac{L}{D} = (M_2^r/M_1^r)\left(1 + \frac{2hb_w}{h_s L_1}\right)^{-1} \tag{1}$$

in which the various floor and beam dimensions are defined in the paper. The live-to-dead load ratio θ is used in the ACI Code to specify live loads for various types of buildings, and it has been adopted by previous researchers as an independent benchmark parameter characterizing the design situation and in terms of which the reliability is computed. Thus, the live-to-dead moment ratio of 2.5 in the discussion corresponds to a live-to-dead load ratio of $\theta = 1.81$ in the paper.

The paper examines the variation of the bending reliability index β in terms of the live-to-dead load ratio θ by keeping fixed cross-section dimensions and changing the steel reinforcement ratio, ρ_s, as needed. On the other hand, the discussion analyzes, for a specific live-to-dead load ratio of $\theta = 1.81$, the variation of the reliability index as a function of the normalized steel reinforcement ratio $\xi = (\rho_s - \rho_{\min})/(\rho_{\max} - \rho_{\min})$ by changing the cross-section dimensions and therefore the relative amount of concrete and steel between designs. In other words, the paper and the discussion look at different "sub-classes" of design situations. The variation of β as a function of the abscissa ξ presented in the discussion would appear as vertical perturbations of the β versus θ curves in the paper. For a given θ value, there would be a range of β values depending on the steel reinforcement ratio adopted; β would decrease as the reinforcement ratio increases. It is the extent of this fluctuation in β that is of interest and it depends on θ and the material strength parameters.

In the paper, the probability of failure (or reliability index) is computed using FORM/SORM and based on the assumption that the probability of brittle failure in bending is negligible. In the discussion, the brittle failure in bending is accounted for and the (global) bending failure probability is obtained through Monte Carlo simulation with sample sizes of 5000 or 15 000. In this closure, a general system approach is used to compute the bending failure probability including both the ductile and brittle failure modes. On neglecting the brittle failure mode due to light reinforcement and referring to the component limit-state functions defined in the paper, the "system" failure domain F can be expressed as

$$F = \{\{(g_2 < 0) \cap (g_4 < 0)\} \cup \{(g_2 > 0) \cap (g_5 < 0)\}\}, \tag{2}$$

and, from the total probability theorem, the bending failure probability is given by

$$p_F = P[g_4 < 0 \mid g_2 < 0]P[g_2 < 0] + P[g_5 < 0 \mid g_2 > 0]P[g_2 > 0].$$

The failure probability of this "system" can be computed by the second-order directional simulation method in which the modal failure surfaces are approximated as hyper-parabolic surfaces fitted at the design point in the SORM analysis, or by the crude Monte Carlo simulation method. Both of these methods are implemented in CALREL and were used for the analysis presented below. In both cases, the simulation stops after 100 000 trials or after the sample coefficient of variation of the failure probability estimate falls below 5 percent (0.05). The above system reliability analysis was performed for different values of the normalized steel reinforcement ratio ξ and for the same live load and material properties as those used in the discussion, namely $f_y = 40$ ksi, $f'_c = 4$ ksi, and $L/D = 1.81$.

As mentioned in the paper, due to lack of experimental data there is a practical problem in choosing the suitable statistics for the model uncertainty factor, B_f, related to the brittle failure mode in bending. Furthermore, Table 2 and Fig. 10 in the paper show large sensitivities of β with respect to the mean and standard deviation of B_f. Although these sensitivity results are

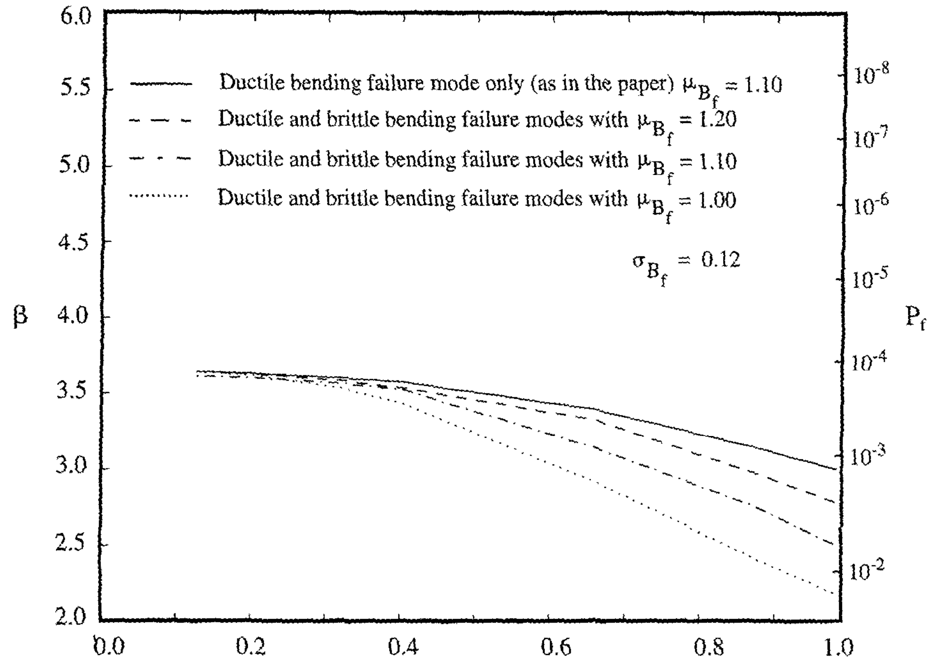

Fig. 1. Bending reliability index β vs. normalized steel reinforcement ratio ξ.

obtained for the ductile bending failure mode, similar results can be expected for the brittle failure mode. Nevertheless, in order to solve the bending reliability problem with the objective of examining the effect of the brittle failure mode on the (global) bending reliability index, the same first- and second-order statistics, μ_{B_f} and σ_{B_f}, are adopted for the ductile and brittle failure modes, and then the sensitivity of the bending reliability index with respect to μ_{B_f} for the brittle failure mode is examined. By varying the dimensions of the beam cross section, different designs are obtained for the same live-to-dead load ratio of $\theta = 1.81$ and the computed reliability index β is plotted against the normalized steel reinforcement ratio ξ as shown in Fig. 1 of this closure. For comparison purposes, Fig. 1 also contains the bending reliability index obtained when accounting for the ductile bending failure mode only. It is observed that the bending reliability indices obtained when neglecting and when accounting for the brittle failure mode both decrease with increasing reinforcement ratio. In the case of the ductile bending failure mode only, the decrease of β with increasing ξ is due to the fact that as the reinforcement ratio increases, the beam flexural strength becomes increasingly sensitive to the concrete strength whose variability is larger than that of the reinforcement steel. The additional decrease of β with respect to ξ when both the ductile and brittle failure modes are considered is explained by the fact that as ξ increases, $P[g_2 > 0]$ in Eq. (2), namely the probability of over-reinforcement, increases and the conditional failure probability $P[g_5 < 0 \mid g_2 > 0]$, that is the probability of bending failure given that the beam is over-reinforced, depends on the "high-variability" concrete strength and is independent of the "low-variability" steel strength. When considering only the ductile bending failure mode, the discusser obtained a constant reliability index with respect to ξ, see Fig. 1 of discussion, thus giving a larger difference in β between the two approaches (ductile only versus ductile and brittle).

Based on the discussion and the additional results presented in this closure, several points are worth mentioning. First, it is found that the difference between the one-mode and two-mode bending reliability indices is negligible for small ξ, say $\xi < 0.4$. Second, most practicing engineers design at 50 percent of the maximum allowable steel reinforcement ratio, corresponding to $0.50(\frac{3}{4}\rho_b) = 0.375\rho_b$ where ρ_b denotes the balanced reinforcement ratio given in Eq. (17) of the paper for a rectangular cross-section, in order to provide adequate beam depth for deflection control. Using the example considered in the discussion, $0.375\rho_b$ corresponds to $\xi = 0.42$. Around this value of ξ, Fig. 1 of both the discussion and the closure show that the contribution of the brittle failure mode on the (global) bending reliability index is practically negligible and that the bending reliability index and corresponding parametric sensitivities obtained by considering only the ductile failure mode are sufficient to provide the designer with a rational safety assessment of the RC beam designed according to the ACI Building Code. Third, to illustrate the sensitivity of β with respect to μ_{B_f}, the mean bias of the analytical strength prediction, the β versus ξ two-mode reliability curve is recomputed for the values of $\mu_{B_f} = 1.00$ and 1.20 and plotted in Fig. 1. It is noticed that the bending reliability index is very sensitive to μ_{B_f}, especially in the range $0.45 < \xi < 1$. Further variations of β can be produced if larger values of σ_{B_f} are considered, which is not shown here. In light of this high sensitivity of the bending reliability index with respect to the first- and second-order statistics of the model uncertainty factor B_f, the bending reliability index obtained by considering both the ductile and brittle failure modes is questionable and should be used with caution until μ_{B_f} and σ_{B_f} are estimated from experimental data on brittle bending failure due to over-reinforcement. Finally, in cases where the member size is limited and a high steel reinforcement ratio cannot be circumvented, the results presented in the discussion and in this closure indicate that the bending failure probability of RC beams satisfying the ACI Building Code requirements may increase significantly. The design code modifications recommended by the discusser might be useful to correct this inconsistency.